# MATHEMATICAL MODELING

## Case Studies from Industry

Edited by

ELLIS CUMBERBATCH
*Claremont Graduate University*

ALISTAIR FITT
*University of Southampton*

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# Introduction

"Technology transfer" has become one of the most well-used phrases of the end of the millennium. The realisation that the worlds of industry and academia cannot fruitfully progress separately has inspired both communities to build strong and mutually beneficial relationships. Often this has meant industry hiring individual professors as consultants, or industry supporting post-doctoral fellows (common in chemistry). An alternative structure has been the utilisation of a quasi-governmental organisation as a go-between, such as NACA/NASA in the US and the Aeronautical Research Council in the UK for work in aeronautics. **Direct** contact between **industry** and the **mathematics community** is more recent, achieving recognition via degree programmes, math-in-industry conferences and journals, all now in a global context. Half of the chapters in this book are products of Study Groups with Industry and Math Clinics, and to some extent the other half derive from similar direct interactions with industry prompted by the successes of these two initiatives.

Study Groups with Industry started in Oxford in 1968 when a small group of applied mathematicians (led by Alan Tayler and Leslie Fox) spent a week problem solving at Oxford University in conjunction with invited representatives from industry. A similar meeting has been held every year since. What *has* changed in recent years is the global nature of this industrial/academic collaboration. In the first 20 or so years Study Groups with Industry only happened in Oxford, and only once a year: by 1999 meetings running on similar lines to the Oxford model had also taken place in Australia, Canada, Denmark, Holland, Indonesia, Mexico, Norway, USA and other locations, in the same year. In many of these ventures domestic applied mathematics societies have encouraged and supported this expansion: pivotal roles have been played in Europe by ECMI (the European Consortium for Mathematics in Industry), in the USA and Canada by SIAM (Society for Industrial and Applied Mathematics) and PIMS (Pacific Institute for the Mathematical Sciences), in Mexico by SMM (Sociedad Matematica Mexicana) and in

Australia by ANZIAM (Australian and New Zealand Industrial and Applied Mathematics).

The *modus operandi* of Study Groups with Industry is now well established: ahead of the meeting there is solicitation from industry for the submission of problems, and probably some discussion regarding appropriateness for a week of brain storming. On the first day of the meeting, the academics, their graduate students, and representatives from industry gather. After each industrialist has described their specific unsolved problem, a room is allotted to each industry, and informal groups are formed. The next three days are spent working intensively on each problem, guided and assisted by the representative from industry. Fierce debate often rages: theories and countertheories come and go, and blackboards are filled with equations. Because of the time constraints that apply, the evenings are often used to work on the problems. Normally, a consensus is eventually reached. On the final day of the meeting, progress on each of the problems is summarised and subsequently a technical problem report is published.

Math Clinics originated at the Claremont Colleges in 1973. The structure here has a longer time frame: problems are solicited over the summer, and work on them is contracted for an academic year. Each problem is addressed over this time period by a team consisting of a faculty supervisor and 4–6 graduate and undergraduate students (one of whom is team leader, a managerial position). The team has the time to do basic research, and during the year makes a number of oral and written reports. Students are assigned grades as for a traditional course.

In both the Study Group and Clinic operations, the involvement of students is important. How problems originate, how industrialists expect them to be modelled, and what they expect from an answer, is valuable experience for students aiming for industrial employment, or starting on research in applied mathematics. Additionally students working on these problems learn the dynamics of group work, and the importance of good oral and writing skills.

What type of problems appear at Study Groups and Clinics? This question is difficult to answer, for the range is huge. Not only do the problems vary enormously in physical and mathematical terms, but the reasons that the industrialist has for bringing a specific problem may vary. Typical "industry questions" all begin "we have a process. . . " but may have a range of endings such as

- ". . . which is well-understood and has worked well for years, but we suspect that it could be made more efficient: how can we optimise it?"

- "... which has worked well for years, but we don't have much idea why. Now we're trying to extend and change it. How does it work?"
- "... which normally works pretty well. Sometimes, though, something goes wrong. Under what circumstances might we expect this to happen, what should we measure to give us warning signs and how might we cure the problem?"
- "... in mind which is new and very promising. Before running expensive and/or dangerous experiments, we'd like to get an idea of how things might turn out."
- "... which works well and we know is safe. But we have to satisfy an external regulatory body to *prove* that it is safe. Can you model the 'worst case' circumstances for us?"
- "... which we have simulated (ourselves, or bought software for). The data and the simulations do not agree. What is going wrong?"

As far as the representatives from industry are concerned, finding the answers to questions of this sort is usually the main priority. There are many other possible advantages of the interaction. The opportunity to spend time discussing their problem with a group of experts is attractive, as is the chance to see what sort of problems other industries have and how they cope with them. Frequently the discussion widens, and other problems may be considered. Finally, there is always the chance to build more longer lasting professional relationships and to recruit promising students.

For the process to be mutually beneficial, everybody concerned must have a good reason for wanting to take part. Some of the benefits for students have been referred to above. What about faculty? It turns out that there are many possible reasons for participating. These include:

- A constant supply of interesting and novel problems which often lead to publications in leading journals
- The chance to form closer relationships with companies which may lead to joint studentships and research contracts
- An opportunity to broaden the range of new problem areas leading to a freshening of the teaching syllabus
- The transfer of mathematics across applications
- A chance to work as part of a team – frequently a novelty for mathematicians.

How much does all of this cost? Considering the possible benefits to both industry and the academics involved, the sums involved are surprisingly small. British Study Groups cost between 10,000–15,000 pounds to run; the money is provided by the participating industries (each problem presenter is asked to contribute about 1000 pounds), and various other organisations such as the London Mathematical Society and (until recently) EPSRC also contribute much-needed revenue. As far as the industrial participants are concerned, the biggest cost is normally that of having their expert in a particular field away from the office for a whole week. For clinics, the financial details are rather more large-scale; each clinic project costs the industry concerned a sum between 35,000 and 45,000 dollars. Although this might sound a lot, for their money the client gets a year's work from a dedicated group of experts: the productivity often equals that of an engineer-year which costs the employer five times as much. Do they consider this money well spent? This is a question that can only truly be answered by the industrial scientists themselves, but it is significant that a large number of projects have been brought to the Claremont clinics by "repeat customers".

Thirteen different problems are considered in this book; all originate from real collaborations with industry. Although some of them are a little difficult to classify, four of the problems are recognisably elliptic, six are parabolic, and two are hyperbolic. This distribution may be said to fairly represent the frequency of each type of problem that is normally encountered at Study Groups.

The order chosen for the articles is related to ordinary and partial differential equation classification. Chapters 1–4 concern models based on essentially elliptic partial differential equations. Chapters 5–10 all involve some form of diffusion, with nonlinear effects, convection and reaction, and finally radiation being successively introduced. The classification of the underlying equations in chapter 11 is less clear; the equations contain both parabolic and hyperbolic features. The concluding two chapters, 12 and 13, address two hyperbolic problems.